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Convective Instability in a Dielectric Fluid Layer Under an AC Electric Field: Effect of Robin Thermal Boundary Conditions

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Abstract— An investigation is carried out to determine the effect of thermal and electrohydrodynamic boundary conditions on the onset of thermal convection in a dielectric fluid layer under the influence of a vertical AC electric field. Thermal boundary conditions are considered at fixed heat flux or constant temperature of lower surface, while upper surface is considered to be Robin thermal boundary conditions. The problem is solved numerically for the thermal or electric Rayleigh number as the eigenvalue using ninth-order Galerkin type of weighted residual technique. It is found that the thermal or electric Rayleigh number marking the onset of electroconvection is greatest for the thermal boundary condition of fixed temperature and least for fixed heat flux condition. It is observed that increase in the critical electric Rayleigh number, is to hasten the onset of electroconvection, while increase in Biot numbers, is to delay the onset of electroconvection. Besides, increase in electric Rayleigh number and decrease in Biot number leads to contraction of cell. Some known results are recovered as special cases from the present study.

Index Terms— Electrohydrodynamic, Dielectric fluid, Electroconvection, heat transfer coefficient, Galerkin technique, Convection cell, AC electric field

1 Introduction

The application of a strong electric field in a poorly conducting fluid can introduce bulk motions. This phenomenon is known as electrohydrodynamics. Electrohydrodynamics has attracted extensive research due to its wide applications in ink-jetting, drug delivery, chemical analysis and dielectric fluid motor [1], [2]. Another potential application of an electric field is to engineering devices ranging from aircraft and space vehicles to micro-fluidic devices [3].

Convection can occur in a dielectric fluid layer even if the temperature gradient is stabilizing and such an instability produced by an AC electric field is called electroconvection or Rayleigh-Bénard electro convection. The study of Rayleigh–Bénard convection in dielectric fluids via the classical linear stability theory has been exhaustively studied by Takashima and Hamabata [4], Oliveri and Atten [5], Stiles [6], Maekawa et al. [7], Stiles et al. [8], Smorodin and Velarde [9], Siddheshwar[10], Othman and Zaki [11], Shivakumara et al. [12], Siddheshwar and Radhakrishna [13] investigated the effect of rotation under AC electric field on the onset of electrohydrodynamic instability in a horizontal couple stress dielectric fluid layer caused by the dielectrophoretic force. Shivakumara et al. [14] investigated the problem effects of velocity and temperature boundary conditions on electrothermal convection in a rotating dielectric fluid layer.

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Vaszquez and Castellanos [17] have solved the problem of charge transport equation in EHD convective flows, in both strong and weak injection regimes numerically by discontinuous Galerkin finite element method. Wu et al. [18] have studied the effect of mobility parameter on the oscillatory electroconvection in dielectric liquids induced by the unipolar charge injection in the case of isothermal. Wu et al. [19] have investigated the hydrodynamic stability of a dielectric fluid layer subjected to strong unipolar injection. Recently, Wu et al. [20] have carried out the linear stability analysis on enhancement of electroconvection in dielectric liquids under unipolar injection model using two-dimensional numerical simulation.

The objective of this paper is to determine analytically the conditions for the onset of electroconvection for a broad variety of thermal and electrohydrodynamic boundary conditions. In particular, consideration is given to the case of arbitrary heat transfer coefficient at either rigid or free upper boundaries corresponding to either fixed temperature or to fixed heat flux condition at a lower rigid boundary. The critical stability parameters are obtained numerically using a ninth-order Galerkin type of weighted residual method. The effects of various physical parameters on the stability of the system are analyzed.

2 Mathematical Formulation

Consider an infinite, incompressible dielectric fluid layer of thickness d with a uniform vertical AC electric field applied across the horizontal layer between the surface $z=0$ and $z=d$. The lower and upper boundaries of the layer are maintained at constant temperatures T_o and $T_1=T_0-\Delta T$ $(\Delta T>0)$ respectively. A Cartesian coordinate system (x, y, z) is chosen with the origin at the bottom of the layer and z- axis normal to the layer. The governing equations for the flow of an incompressible dielectric fluid are:

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} \right] = -\nabla p + \rho_0 [1 - \alpha (T - T_0)] \vec{g} + \mu \nabla^2 \vec{q} + \vec{f}_e \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa \nabla^2 T \tag{3}$$

Assuming the free charge density is negligibly small, the relevant Maxwell's equations are

$$\nabla \cdot (\varepsilon \vec{E}) = 0 \tag{4}$$

$$\nabla \times \vec{E} = 0 \text{ or } \vec{E} = -\nabla \phi \tag{5}$$

$$\varepsilon = \varepsilon_0 [1 - \eta (T - T_0)] \tag{6}$$

where $\vec{q} = (u, v, w)$ the velocity vector, $\vec{g} = (0, 0, -g)$ the gravitational acceleration, T the temperature, p the pressure, ρ_0 the density at reference temperature $T = T_o$, κ the effective thermal diffusivity, μ the fluid viscosity, $\vec{E} = (0, 0, E_z)$ the applied AC electric field, $\alpha (>0)$ the thermal expansion coefficient, η (>0) the analog for dielectric constant of thermal expansion coefficient, ε_0 the dielectric constant at reference temperature $T = T_o$, ϕ the root mean square velocity of the electric potential and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ the Laplacian operator. The last term of Eq.(2) is the force of electrical origin which is of the form:

$$\vec{f}_e = \rho_e \vec{E} - \tfrac{1}{2} (\vec{E} \cdot \vec{E}) \nabla \varepsilon + \tfrac{1}{2} \nabla \left[\rho \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T \vec{E} \cdot \vec{E} \right] \tag{7}$$

Following the standard linear stability analysis procedure and noting that the principle of exchange of stability holds good in equation (7), the first term stands for coulomb force is neglected since there are no free charges. The second term is dielectro phoretic force and third term is electrostrictive force which is a conservative vector and can be convenient combined with static pressure. The stability equations in dimensionless form can then be shown to be

$$\begin{bmatrix} (D^2 - a^2)^2 & -a^2 (R_t + R_e) & a^2 R_e D \\ 1 & D^2 - a^2 & 0 \\ 0 & -D & D^2 - a^2 \end{bmatrix} \begin{bmatrix} W \\ \Theta \\ \Phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{8a,b,c}$$

where $D \equiv d/dz$ the differential operator, $a = \sqrt{\ell^2 + m^2}$ the horizontal wave number, $R_t = \alpha g \Delta T d^3 / \nu \kappa$ the thermal Rayleigh number and $R_e = \eta^2 \varepsilon_0 E_0^2 (\Delta T)^2 d^2 / \mu \kappa$ the electric Rayleigh number. Equations (8a,b,c) forms an eigenvalue problem for R_t or R_e and is to be solved using various types of boundary conditions. The following boundary combinations are considered:

(i) Rigid-rigid boundaries:

$W = DW = \Phi = 0$, $\Theta = 0$ or $D\Theta = 0$ at $z = 0$ (9a)

$W = DW = \Phi = D\Theta + Bi\,\Theta = 0$ at $z = 1$ (9b)

(ii) Lower rigid-upper free boundaries:

$W = DW = \Phi = 0$, $\Theta = 0$ or $D\Theta = 0$ at $z = 0$ (10a)

$W = D^2 W = D\Theta + Bi\,\Theta = D\Phi = 0$ at $z = 1$ (10b)

(iii) Free-free boundaries:

$W = D^2 W = D\Phi = 0$, $\Theta = 0$ or $D\Theta = 0$ at $z = 0$ (11a)

$W = D^2 W = D\Theta + Bi\,\Theta = D\Phi = 0$ at $z = 1$. (11b)

In equations (9b), (10b) and (11b), the Biot number $Bi = hd/k_t$ is the ratio of the rate of heat removal from the interface to the environment to the rate of heat supply to the interface from the bulk of a fluid due to thermal conduction at the upper boundary. Increase in Bi from 0 to ∞ means a change in the thermal condition at the upper boundary from "fixed heat flux" condition to "constant temperature" condition.

3 Method of Solution

Equations (8a,b,c) together with boundary conditions (9)–(11) are analytically intractable and therefore one has to implement a numerical technique by applying the ninth-order Galerkin type of weighted residual technique to obtain the critical stability parameter. Accordingly, the dependent variables are written as a series of linearly independent functions (basis functions) :

$$W = \sum_{i=1}^{9} A_i W_i \;, \; \Theta = \sum_{i=1}^{9} B_i \Theta_i \;, \; \Phi = \sum_{i=1}^{9} C_i \Phi_i \;, \tag{12}$$

where A_i, B_i and C_i are constants and the basis functions W_i, Θ_i and Φ_i will be represented by the power series satisfying the respective boundary conditions. Substituting equation (12) into equations (8a,b,c), multiplying the resulting momentum equation (8a) by $W_j(z)$, energy equation (8b) by $\Theta_j(z)$, electric potential equation (8c) by $\Phi_j(z)$; performing the integration by parts with respect to z between z = 0 and z = 1 and using the boundary conditions given by (9)-(11), we obtain the system of algebraic equations has a non-trivial solution if and only if

$$\begin{vmatrix} E_{ji} & F_{ji} & G_{ji} \\ I_{ji} & J_{ji} & 0 \\ 0 & K_{ji} & L_{ji} \end{vmatrix} = 0 \tag{13}$$

where

$E_{ji} = \langle D^2 W_j D^2 W_i \rangle + 2a^2 \langle DW_j DW_i \rangle + a^2 \langle W_j W_i \rangle$

$F_{ji} = -a^2 (R_t + R_e) \langle W_j \Theta_i \rangle$

$G_{ji} = -a^2 R_{ea} \langle DW_j \Phi_i \rangle$

$I_{ji} = \langle \Theta_j W_i \rangle$

$J_{ji} = -\langle D\Theta_j D\Theta_i \rangle - a^2 \langle \Theta_j \Theta_i - Bi\,\Theta_j(1) \Theta_i(1) \rangle$

$K_{ji} = -\langle D\Theta_j \Phi_i \rangle$

$L_{ji} = -\langle D\Phi_j D\Phi_i \rangle - a^2 \langle \Phi_j \Phi_i \rangle$

The eigenvalue has to be extracted from the above characteristic equation. For this, we select the trial functions satisfying the appropriate boundary conditions (i) rigid-rigid, (ii) rigid-free and (iii) free-free respectively as

(i) $W_i = (z^4 - 2z^3 + z^2) T_i^*$, $\Theta_i = (z - z^2/2) T_i^*$, $\Phi_i = (z - z^2) T_i^*$ (14)

(ii) $W_i = (2z^4 - 5z^3 + 3z^2) T_i^*$, $\Theta_i = (z - z^2/2) T_i^*$, $\Phi_i = (z - z^2/2) T_i^*$ (15)

(iii) $W_i = (z^4 - 2z^3 + z) T_i^*$, $\Theta_i = (z - z^2/2) T_i^*$, $\Phi_i = (z^3 - 3z^2/2) T_i^*$ (16)

where T_i^*'s are Chebyshev polynomials ($i = 0, 1, 2, \ldots$).of the second kind. The above trial functions satisfy all the boundary conditions except the natural one, namely $D\Theta + Bi\,\Theta = 0$ at $z = 1$ but the residual is included from the differential equations. Substituting equations (14)-(16) in equation (13) and expanding the determinant leads to the characteristic equation giving the Rayleigh number R_t (or R_e) as a function of the wave number a as well as other parameter Bi. The critical thermal

Rayleigh number R_{tc} or critical thermal electric Rayleigh number R_{ec} is obtained by minimizing R_t or R_e respectively with respect to the wave number ' a ' for different values of Biot number, Bi. It is apt to note here that the ninth-order Galerkin procedure ($i = j = 9$) ensures convergence for an accuracy of 10^{-4} in the calculated eigen values.

4 Results and Discussion

The classical linear stability analysis has been carried out to investigate the effect of Robin Thermal boundary conditions on the onset of electroconvection in dielectric fluid heated from below in the presence of a vertical AC electric field. Attention is focused on the different forms of boundary conditions keeping in mind the laboratory and engineering problems. The lower boundary of the dielectric fluid layer is assumed to be rigid/free at fixed temperature or at fixed heat flux, while the upper boundary is rigid or stress-free boundary conditions. The critical stability parameters are computed numerically by Galerkin technique. The critical thermal Rayleigh number (R_{tc}) or critical thermal electric Rayleigh number (R_{ec}) and the corresponding critical wave number (a_{ec}) are used to characterize the stability of the system.

To confirm, the results by applying numerical procedure and to make a comparison with some existing results (see Table 1-2), the linear theory analysis results are compared very well with those of Sparrow et al. [15] for several values of Bi with fixed values of R_{ec} =0. To assure the validity of solution procedure, the critical results [R_{ec} , a_{ec}] are compared with those of Roberts [16] for selected values of R_{tc} in the absence of heat transfer coefficient (Bi =0).

TABLE 1

Comparison of R_{tc} and a_{ec} for lower boundary rigid at fixed temperature with different values of Bi in the absence of electric Rayleigh number

	Lower boundary rigid at fixed temperature							
	Upper boundary free				Upper boundary rigid			
	Sparrow et al [15]		Present study		Sparrow et al [15]		Present study	
B_i	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c	R_{tc}	a_c
0	669.001	2.09	668.98	2.086	1295.781	2.55	1295.78	2.552
0.1	682.361	2.115	682.36	2.116	1309.545	2.58	1309.54	2.582
0.3	706.365	2.17	706.39	2.169	1334.149	2.64	1334.08	2.632
1	770.569	2.3	770.57	2.293	1398.508	2.75	1398.51	2.751
3	872.506	2.46	872.53	2.452	1497.594	2.90	1497.57	2.901
30	1055.345	2.65	1055.5	2.648	1667.102	3.08	16671	3.084
100	1085.893	2.67	1085.9	2.672	1694.573	3.11	1694.57	3.106
∞	1100.657	2.68	1100.5	2.682	1707.765	3.12	1707.76	3.116
Lower boundary rigid at fixed heat flux								
0	320.00	0	320	0.648	720.000	0.71	720	0.679
0.1	381.665	1.015	381.665	1.015	807.676	1.23	807.676	1.228
0.3	428.29	1.3	428.29	1.299	869.231	1.57	869.208	1.557
1	513.792	1.64	513.79	1.644	974.173	1.94	974.172	1.943
3	619.666	1.92	619.666	1.921	1093.744	2.24	1093.74	2.242
30	780.240	2.18	780.237	2.176	1259.884	2.51	1259.91	2.511
100	804.973	2.2	804.972	2.203	1284.263	2.53	1284.28	2.539
∞	816.748	2.21	816.744	2.215	1295.781	2.55	1295.781	2.552

TABLE 2
COMPARISON OF R_{tc} AND a_c FOR DIFFERENT VALUES OF R_e FOR $Bi = 0$

Roberts [16]			Present study	
R_t	R_e	a_c	R_e	a_c
-1000	3370.077	3.2945	3370.077	3.29446
-500	2749.868	3.2598	2749.868	3.25983
0	2128.696	3.2260	2128.696	3.22596
500	1506.573	3.1929	1506.573	3.19287
1000	883.517	3.1606	883.517	3.16059
1707.762	0	3.1162	0	3.11621

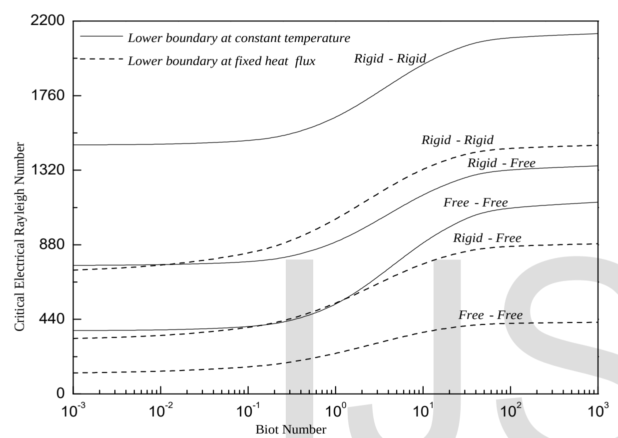

Fig. 1. Critical electric Rayleigh number R_{ce} as a function of Biot number Bi for different boundaries when $R_t = 0$

In figures 1-3, three solid curves correspond to lower boundary at constant temperature and three dotted curves correspond to lower boundary at fixed heat flux condition. The plot of R_{ec} against Bi in the absence of buoyancy force for different forms of thermal and hydrodynamic boundary conditions (R-R, R-F, F-F) as shown in Figure 1. It shows that they are bridging the space between the results of fixed surface heat flux and constant temperature at the lower surface when increasing the heat transfer coefficient Bi (i.e., Biot number). Clearly, fixed heat flux condition at lower surfaces advances electro convection compared to constant temperature. Figure 1 reveals that the linear stability analysis can be expressed in terms of R_{ec}, the system with rigid-rigid surface is most stable compared to the system with free-free surfaces. In the figure, critical values of R_{ec} initially increases slowly and then increases quickly towards an asymptotic value for R-R, R-F and F-F respectively at R_{ec} 2126.26, 1304.54 and 1036.49 in the case of lower surface at constant temperature and also at R_{ec} 1415.71, 842.30 and 402.69 in the case of lower surface at fixed heat flux for corresponding to minimum of each minimum of each curve increases with Bi, this means that the dielectric fluid layer under AC electric field becomes more stable with increase of Bi. On the upper free surface, for small values of Bi, these perturbations are very prone to heat transfer coefficient and for large values of Bi, these can be regarded as an imposed constant temperature that causes R_{ec} to approach this asymptotic value.

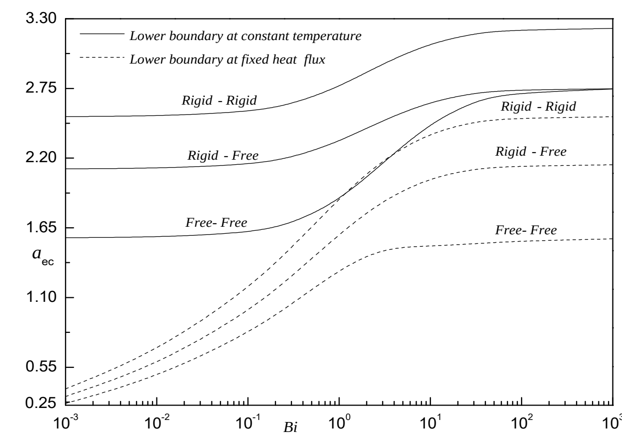

Fig. 2. Critical electric wave number a_{ec} as a function of Biot number Bi for different boundaries when $R_t = 0$

The findings reported in figure 2 for pure electroconvection essentially reiterate the observation made in the context of classical Rayleigh-Bénard convection. The following general results are true:

(i) $(a_{ec})_{Bi=0} > (a_{ec})_{Bi=finite} > (a_{ec})_{Bi\to\infty}$

(ii) $(a_{ec})_{rigid-rigid} > (a_{ec})_{rigid-free} > (a_{ec})_{free-free}$.

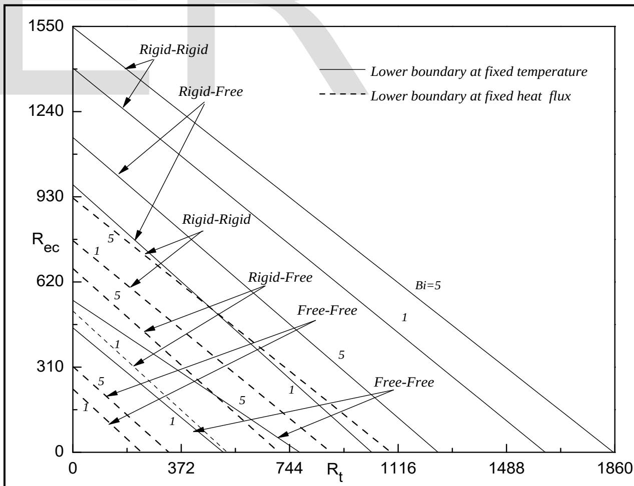

Fig. 3. Variations of R_{ce} as a function of R_{tc} for two values of Bi with different boundary conditions

To know the simultaneous presence of buoyancy and electric forces on the stability of the system, the locus of R_{ec} and R_{tc} is exhibited in Figure 3 for different boundary conditions and two values of Biot number Bi (= 1 and 5). From the figure, it is observed that there is a strong coupling between the critical electric Rayleigh number R_{ec} and the critical thermal Rayleigh number R_{tc}. If there is increase in the value of one, then there is a decrease in the value of other. On seeing the result of figure 3, it is pretty obvious that increasing the strengths of electric field leads to destabilization of the Rayleigh-Bénard configuration. This result is true for all boundary conditions considered. The observed effect of Bi seen in Figure 3 may be attributed to the fact that with increasing Bi, the thermal disturbances can easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the top surface and hence higher heating is required to make the system unstable.

5 Conclusions

The effect of vertical AC electric field on the onset of electroconvection in a dielectric fluid layer heated uniformly from below is investigated for a variety of thermal and electrohydrodynamic boundary conditions. The effect of AC electric field strength, the heat transfer coefficient is enhanced to various types of boundary conditions. Thus their effect is to delay the onset of electroconvection. The results for different thermal and hydrodynamic boundary conditions differ only quantitatively. The system was more stable when both boundaries are rigid while free-free boundaries are least stable. For fixed heat flux condition at the upper boundary advances electroconvection compared to constant temperature condition. The critical wave number increases with an increase in the AC electrical Rayleigh number and the Biot number. Thus their effect is to contract the size of convection cells.

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